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Total No. of Questions : 09

B.Tech. (Sem.–2)

ENGINEERING MATHEMATICS-II

Subject Code : AM-102 (2005 - 2010 Batch)

Paper ID : [A0119]

Time : 3 Hrs. Max. Marks : 60

INSTRUCTION TO CANDIDATES :

1. SECTION-A is COMPULSORY consisting of TEN questions carrying TWO marks each.
2. SECTION-B contains FIVE questions carrying FIVE marks each and students has to attempt any FOUR questions.
3. SECTION-C contains THREE questions carrying TEN marks each and students has to attempt any TWO questions.

SECTION-A

1. Write short notes on :

   a) Define rank of a matrix. What could be the maximum value of a rank of a 3×4 matrix?

   b) Derive the condition for the linear transformation Y = AX to be orthogonal, where A is a square matrix.

   c) What is necessary and sufficient condition for a differential equation $M dx + N dy = 0$ to be exact ?

   d) Find the particular integral of the differential equation

      $(D^3+4D)y = \sin 2x.$

   e) Consider an electric circuit containing an inductance L and capacitance C. Let i be the current and q the charge in the condenser plate at any time t. Write down the differential equation of charge for this circuit. What is the nature of this differential equation ?

   f) Show that the vector $3y^4 z^2 \hat{i} + 4x^3 z^2 \hat{j} + 3x^2 y^2 \hat{k}$ is solenoidal.

   g) State Green's theorem in plane.

h) Show that the vector field $\vec{F} = (x^2 - y^2 + x)\hat{i} - (2xy + y)\hat{j}$ is irrotational.

i) Define the terms 'Exhaustive events' and 'Mutually exclusive events'.

j) Write a short note on 'objectives of sampling'.

SECTION-B

2. a) Reduce the following matrix to normal form and hence find its rank:

$$A = \begin{bmatrix} 8 & 1 & 3 & 6 \\ 0 & 3 & 2 & 2 \\ -8 & -1 & -3 & 4 \end{bmatrix}$$

b) Test the following system of equations for consistency and solve.

$2x - 3y + 7z = 5$; $3x + y - 3z = 13$; $2x + 19y - 47z = 32$.

3. Find complete solutions of the following differential equations:

a) $(x^2y^2 + xy + 1)ydx + (x^2y^2 - xy + 1)xdy = 0$

b) $p^2 + 2py \cot x = y^2$

4. a) Find a complementary function and particular integral of the differential equation

$$\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = e^{e^x}$$

b) Find complete solution of the differential equation :

$$x^3 \frac{d^3y}{dx^3} + 2x^2 \frac{d^2y}{dx^2} + 2y = 10\left(x + \frac{1}{x}\right)$$

5. An elastic string of natural length 'a' is fixed at one end and a particle of mass 'm' hangs freely from the other end. The modulus of elasticity is 'mg'. The particle is pulled down a further distance 'l' below its equilibrium position and released from rest. Show that the motion of the particle is simple harmonic and find the periodicity.

SECTION-C

6. a) Find the directional derivative of $\varphi(x, y, z) = x^2yz + 4xz^2$ at the point $(1, -2, -1)$ in the direction of the vector $2\hat{i} - \hat{j} - 2\hat{k}$.

   b) If $\vec{F} = (5xy - 6x^2)\hat{i} + (2y - 4x)\hat{j}$ evaluate $\int_C \vec{F} \cdot d\vec{R}$ along the curve C in the XY-plane, $y = x^3$ from the point (1,1) to (2,8).

7. a) Verify Green's theorem for $\int_C [(3x - 8y^2)dx + (4y - 6xy)dy]$ where C is the boundary of the region bounded by $x = 0$, $y = 0$ and $x + y = 1$.

   b) Apply Stoke's theorem $\oint_C (yzdx + zxdy + xydz)$ where C is the curve $x^2 + y^2 = 1$, $z = y^2$.

8. a) Show that the function defined as under is a density function

$$f(x) = \begin{cases} e^{-x}, & x \geq 0 \\ 0, & x < 0 \end{cases}$$

   Determine the probability that the variate having this density will fall in the interval (1, 2). Also find the cumulative probability function F(2).

   b) Fit a parabola $y = a + bx + cx^2$ to the following data :

x :	2	4	6	8	10
y :	3.07	12.85	31.47	57.38	91.29.

9. a) A die was thrown 9000 times and a throw of 5 or 6 was obtained 3240 times. On the assumption of random throwing, do the data indicate an unbiased die ?

   b) The nine items of a sample have the following values: 45, 47, 50, 52, 48, 47, 49, 53, 51. Does the mean of these differ significantly from the assumed mean of 47.5 ?